Computational Ocean Acoustics

(Ray Theory)

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Presentation Outline

- Introduction.
- Mathematical Derivation.
- Mathematical properties.
- WKB method.
- Wavenumber integration technique .

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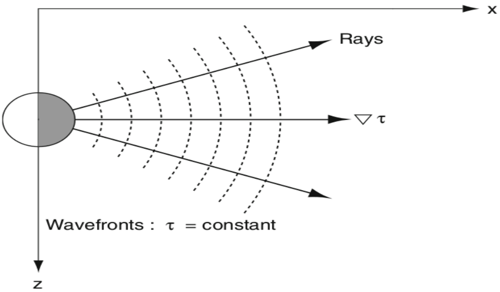

- Ray is defined as normal to a wavefront.
- Ray is characterized by the angle and trajectory.

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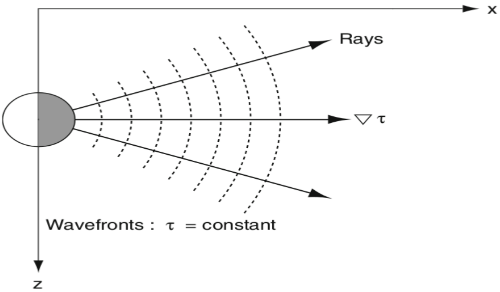

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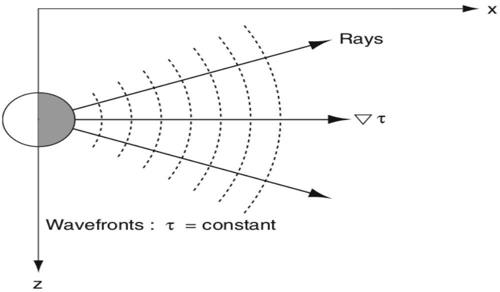

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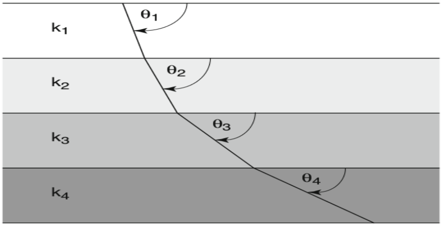

- Ray angle can be found using snell's law.

$$k(z_i)cos\theta(z_i) = k(z_{i+1})cos\theta(z_{i+1})$$

- The ray trajectory can be found using the figure above

$$\frac{dr}{dz} = cot\theta(z) \qquad \text{and} \qquad r(z) = \int_{z_0}^{z_r} cot\theta(z)dz$$

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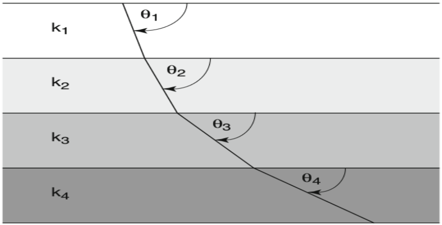

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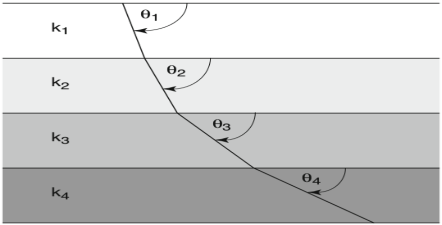

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Ray series solution of wave equation

- Starting with helmholtz equation in Cartesian coordinates

$$\nabla^2 p + \frac{\omega^2}{c^2(x)} p = \delta(x - x_0)$$

- In the ray theory, the solution will be of the ray series form

$$p(x) = e^{i\omega\tau(x)} \sum_{j=0}^{\infty} \frac{A_j(x)}{(i\omega)^j}$$

- Taking the derivative and computing the divergence we have

$$\nabla^2 p = e^{i\omega\tau}[-\omega^2 |\nabla\tau^2| + i\omega\nabla^2\tau] \sum_{j=0}^{\infty} \frac{A_j}{(i\omega)^j}$$

$$+e^{i\omega\tau(x)} \left\{ 2i\omega\nabla\tau \cdot \sum_{j=0}^{\infty} \frac{\nabla A_j}{(i\omega)^j} + \sum_{j=0}^{\infty} \frac{\nabla^2 A_j}{(i\omega)^j} \right\}$$

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Eikonal equation

- Eikonal equation is used to trace the acoustic rays and find the phase of the pressure field.

$$|\nabla\tau|^2 = \frac{1}{c^2(x)}$$

- Ray coordinates are used to linearize the differential equation.
- Considering $\nabla\tau$ perpendicular to the wavefront, the ray trajectory $x(s)$ is defined as.

$$\frac{dx}{ds} = c\nabla\tau \quad \text{and} \quad \left|\frac{dx}{ds}\right|^2 = c^2|\nabla\tau|^2$$

- The eikonal equation can be represented in terms of $c(x)$ by doing some manipulations.

$$\frac{d}{ds}\left(\frac{1}{c}\frac{dx}{ds}\right) = -\frac{1}{c^2}\nabla c$$

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$$\frac{d}{ds}\left(\frac{1}{c}\frac{dx}{ds}\right) = -\frac{1}{c^2}\nabla c$$

Eikonal equation

- In cylindrical coordinates (r, z), these ray equations will be given in first-order form as.

$$\frac{dr}{dz} = c\xi(s), \qquad \frac{d\xi}{ds} = -\frac{1}{c^2}\frac{\partial c}{\partial r}$$
$$\frac{dz}{ds} = c\zeta(s) \qquad \frac{d\zeta}{ds} = -\frac{1}{c^2}\frac{\partial c}{\partial z}$$

- In order to completely specify the equation, the initial conditions are used to specify the source position (r_0, z_0) and take-off angle θ_0.

$$r = r_0, \qquad \xi = -\frac{cos\theta_0}{c(0)}$$
$$z = z_0, \qquad \zeta = -\frac{sin\theta_0}{c(0)}$$

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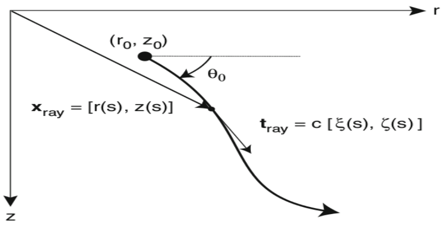

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Phase of the pressure field

- The phase of the pressure field is obtained from the eikonal equation in ray coordinate system

$$\nabla\tau \cdot \nabla\tau = \frac{1}{c^2}$$
$$\nabla\tau \cdot \frac{1}{c}\frac{dx}{ds} = \frac{1}{c^2}$$
$$\frac{d\tau}{ds} = \frac{1}{c}$$

- This is the eikonal equation in ray coordinate s. Linearizing the PDE we have

$$\tau(s) = \tau(0) + \int_0^s \frac{1}{c(s')} ds'$$

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Ray amplitudes and Jacobian

- The amplitude of the pressure field is obtained from the transport equation in ray cooridinate system

$$2\nabla\tau \cdot \nabla A_0 + (\nabla^2\tau)A_0 = 0$$

- Rays are defined as being perpendicular to the wavefront so

$$\frac{2}{c}\frac{dx}{ds} \cdot \nabla A_0 + (\nabla^2\tau)A_0 = 0$$

- The first term represents the directional derivative along the ray path so

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- $\nabla^2\tau$ can be calculated using the Jacobian

$$\nabla^2\tau = \frac{1}{J}\frac{d}{ds}\left(\frac{J}{c}\right)$$

- Putting this value and integrating we have

$$A_0(s) = A_0(0)\left|\frac{c(s)J(0)}{c(0)J(s)}\right|^{1/2}$$

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- Initial condition is found using the canonical problems method.
- Considering the point source in an infinite homogenous media as canonical problem

$$p^0(s) = \frac{e^{i\omega s/c_0}}{4\pi s}$$

- where s is the distance from the source and $c_0 = c|_{s=0}$.
- The amplitude and the phase associated with the solution are.

$$A^0(s) = \frac{1}{4\pi s} \qquad \tau^0(s) = \frac{s}{c_0}$$

$$A^0(0) = \infty \qquad \tau(0) = 0 \qquad (1)$$

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Initial Conditions

- For A_0 the intial condition is computed for $A_0(0)|J(0)|^{1/2}$
- The Jacobian in the homongenous medium turns out to be

$$J(s) = -s^2 cos\theta_0$$

- The initial condition will be

$$\lim_{s \to 0} A(s)|J(s)|^{1/2} = \frac{1}{4\pi}|cos\theta_0|^{1/2}$$

- Putting this in $A_0(s)$ and then obtaining the complete pressure field,

$$p(s) = \frac{1}{4\pi}\left|\frac{c(s)cos\theta_0}{c(0)J(s)}\right|^{1/2} e^{i\omega \int_0^s \frac{1}{c(s')}ds'}$$

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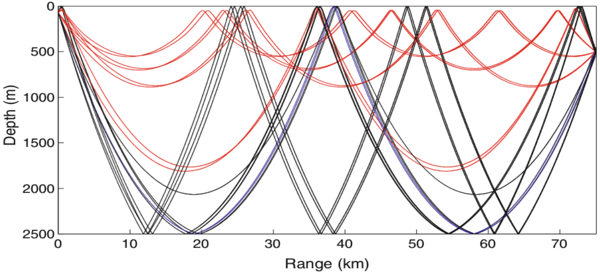
Coherent Transmission Loss



- The intensity at any point in the pressure field is the sum of the contribution of each eigen ray

$$p^{(C)}(r,z) = \sum_{j=1}^{N(r,z)} p_j(r,z)$$

- The number of contributing eigenrays varies with the range and source-receiver position.

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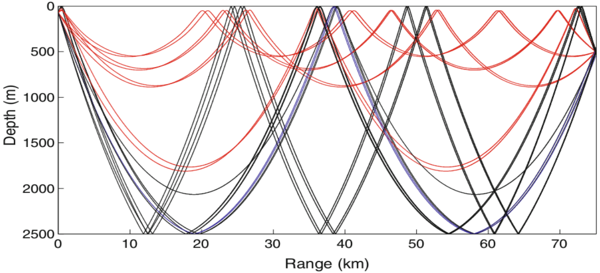

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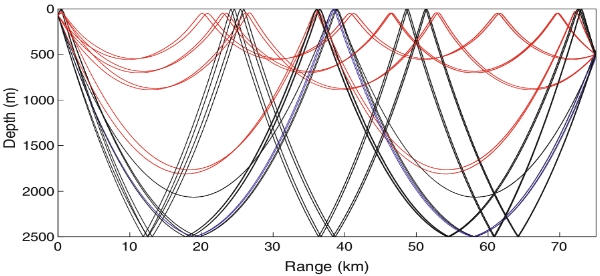

- The intensity at any point in the pressure field is the sum of the contribution of each eigen ray

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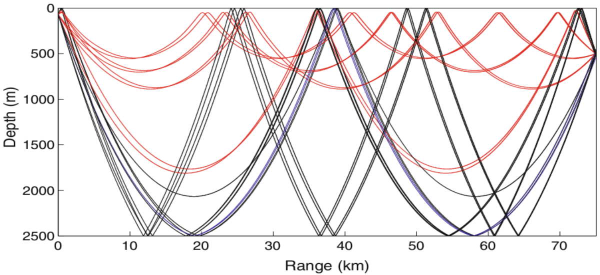

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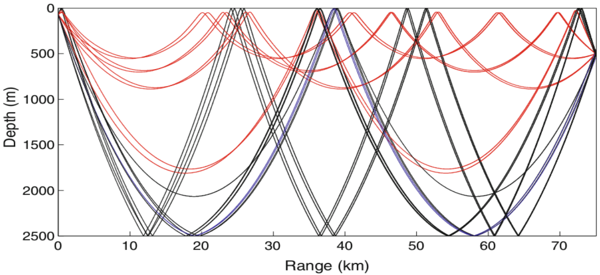

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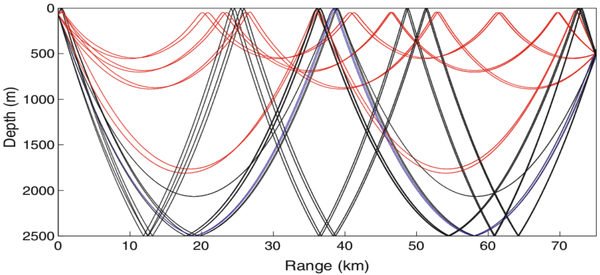

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Incoherent Transmission Loss

- Ray methods are used for high frequency problems.
- At high frequencies, the pressure field is very sensitive to the environmental factors and detailed knowledge of the enviroment is not available all the time.
- In this case incoherent calculations are considered where the phase of the pressure associated with each ray is ignored.

$$p^{(I)}(r,z) = \left[\sum_{j=1}^{N(r,z)} |p_j(r,z)|^2 \right]^{1/2}$$

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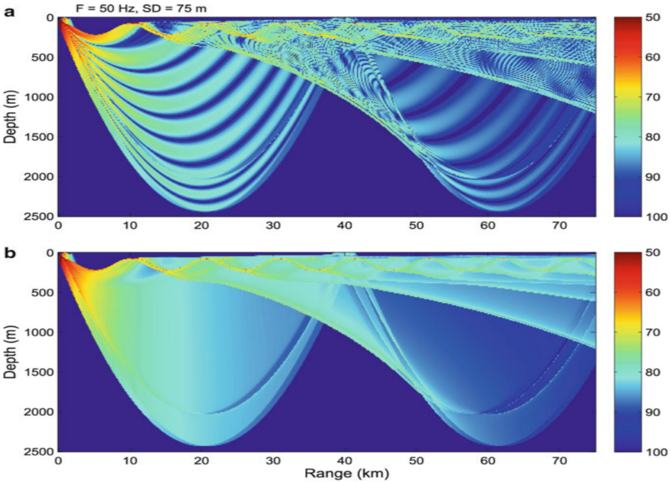

Semicoherent Transmission Loss

- Compromises between the coherent and non-coherent solutions.
- Informal and partially empirical based techniques.

$$p^{(S)}(r,z) = \left[\sum_{j=1}^{N(r,z)} S(\theta_0)|p_j(r,z)|^2\right]^{1/2}$$

- $S(\theta_0)$ is a shading function which weights the amplitude of the rays as a function of its take-off angle.
- Appropriate for directional sources.

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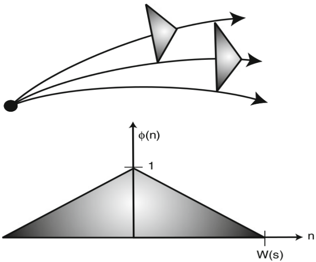

- How to calculate the field in between the rays ?
- Interpolate from the ray grid by constructing a beam around each ray.
- The amplitude of the rays varies linearly on both side of the ray.
- The halfwidth is chosen so that the beam vanishes at the location of its neighboring ray.

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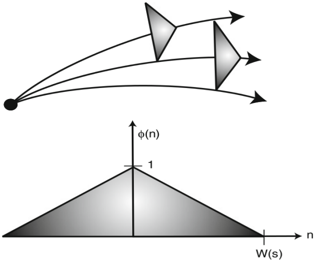

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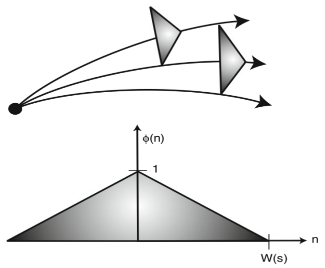

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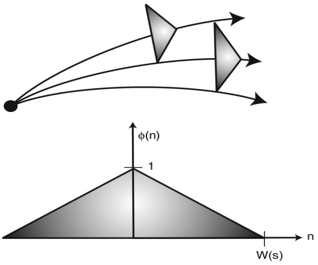

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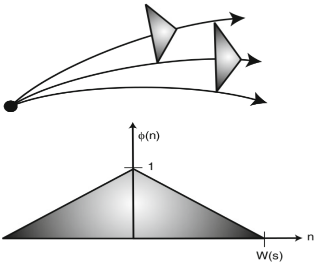

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Treatment of attenuation

- Volumetric attenuation can be included by adding an imaginary part to the sound speed.
- Complicates the process of finding the eigenrays.
- To simplify, we can neglect the imaginary part of the sound speed and add a loss corresponding to the path length of the real rays.
- Eikonal equation is given by.

$$c^2(x)|\nabla\tau|^2 = 1$$

- Perturbed sound speed is given by.

$$c = c_0 + \epsilon c_1 + \cdots$$

- and seeking solution of the form.

$$\tau = \tau_0 + \epsilon\tau_1 + \cdots$$

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- Comparing $O(\epsilon)$ term.

$$c_0^2(\nabla\tau_0)(\nabla\tau_1) + c_0c_1|\nabla\tau_0|^2 = 0$$

- After some manipulations we have .

$$\tau_1(s) = -\int_0^s \frac{c_1(s')}{c_0^2(s')}ds'$$

- If the perturbation is due to loss α, it introduces an imaginary part in the sound speed.

$$ic_i \simeq -i\alpha\frac{|c_r^2|}{\omega}$$

- Putting this value the complete pressure field becomes.

$$p(s) = \frac{1}{4\pi}\left|\frac{c(s)cos\theta_0}{c(0)J(s)}\right|^{1/2} e^{i\omega\int_0^s \frac{1}{c(s')}ds'} e^{-i\int_0^s \alpha(s')ds'}$$

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WKB Method

- Application of ray theory to one dimensional problems.
- The derivation of WKB approximation proceeds the same way except all vectors equations becomes scalar with single variable z.

$$p(z) = e^{i\omega\tau(z)} \sum_{j=0}^{\infty} \frac{A_j(z)}{(i\omega)^j}$$

- Similarly the eikonal and transport equations become.

$$\left|\frac{d\tau}{dz}\right|^2 = \frac{k_z^2(z)}{\omega^2}$$

$$2\frac{d\tau}{dz}\frac{dA_0}{dz} + \frac{d^2\tau}{dz^2}A_0 = 0$$

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WKB Method

- Solving the eikonal equation we have

$$\tau(z) = \pm\frac{1}{\omega}\int k_z(z)dz$$

- Using this result, the first transport equation becomes

$$2k_z(z)\frac{dA_0}{dz} + \frac{dk_z}{dz}A_0 = 0$$

- Implying

$$A_0(z) = \frac{B}{\sqrt{k_z(z)}}$$

- Putting is all together we have the pressure field,

$$p(k_r, z) \simeq \frac{Be^{\pm\int_{z_0}^{z} k_z(z')dz'}}{\sqrt{k_z(z)}}$$

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- WKB solution breaks down in the vicinity of the turning points.
- 2D ray result gives the field at a given point in terms of contribution of a finite number of eigenrays contribution.
- WKB solution represents a single spectral component and the field at any point is obtained by summing up infinite number of contributions.
- Spectral integral is asymptotically dominated by particular points in the k_r spectrum.
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- Take-off angle of eigen rays is different for different source receiver positions, similarly different part of the spectral integral produce dominant contribution depending on the source recevier depths.

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- Numerical implementation of the integral transform technique for horizontally stratified media.
- The final solution is in the form of spectral wavenumber integral.
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Integral Transform solution

- Considering cylinderical coordinate system (r, φ, z) with z-axis passing through the source, the field is independent of the azimuthal angle φ.
- The acoustic field in layer m containing the source can be expressed in terms of displacement potential $\psi_m(r, z)$ satisfying the Helmholtz equation.

$$[\nabla^2 + k_m^2(z)]\psi_m(r, z) = f_s(z, \omega)\frac{\delta(r)}{2\pi r} \quad \text{where} \quad k_m(z) = \frac{\omega}{c(z)}$$

- The solution of the wave equation will be a combination of green functions.

$$\psi_m(k_r, z) = \widehat{\psi_m}(k_r, z) + A_m^+(k_r)\psi_m^+(k_r, z) + A_m^-(k_r)\psi_m^-(k_r, z)$$

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Homogeneous Fluid Layers

- In the absence of a source, the total field can be represented as a combination of an upgoing and downgoing canonical wave.

$$\phi(r,z) = \int_0^{\infty} \left[A^- e^{-ik_z z} + A^+ e^{-ik_z z}\right] J_0(k_r r) k_r dk_r$$

- To evaluate the boundry conditions vertical displacement $w(r,z)$ and normal stress σ_{zz} is used.

$$w(r,z) = \frac{\partial \phi}{\partial z} \qquad \sigma_{zz} = -\rho\omega^2 \phi(r,z)$$

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