Computational Ocean Acoustics

(Wave Propagation Theory)

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Presentation Outline

- Wave Equation.
- Homogeneous Media.
- Layered Media and waveguides.
- Reflection and transmission co-efficients.
- Ideal Fluid waveguide.
- Deep Ocean waveguide and WKB approximation.

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Wave equation

- **Continuity Equation:** Net change in the mass due to its flow through an element is equal to the changes in the density of the mass of element.

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho v)$$

- **Euler's Equation:** Force equals mass time acceleration.

$$\rho \left[\frac{\partial v}{\partial t} + v \cdot \nabla v \right] = -\nabla p$$

- **State Equation:** Relationship between the change in density and a change in pressure.

$$p = p(\rho, S) \text{ where } S \text{ is the entropy}$$

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Linear Wave equation

- Assuming that each physical quantity is a function of a steady-state, time-independent value and a small fluctuation term

$$p = p_0 + p'$$
$$\rho = \rho_0 + \rho'$$
$$v = v_0 + v'$$

- The linearized equations are

$$\frac{\partial \rho'}{\partial t} = -\nabla \cdot (\rho_0 v)$$
$$\frac{\partial v}{\partial t} = -\frac{1}{\rho_0} \nabla p'(\rho)$$
$$\frac{\partial p'}{\partial t} = c^2 \left(\frac{\partial \rho'}{\partial t} + v \cdot \nabla \rho_0 \right)$$

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Different forms of Wave equation

- Wave equation for pressure

$$\rho\nabla\cdot\left(\frac{1}{\rho}\nabla p\right)-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}=0$$

- Wave equation for particle velocity

$$\frac{1}{\rho}\nabla(\rho c^2\nabla\cdot v)-\frac{\partial^2 v}{\partial t^2}=0$$

- Wave equation for velocity potential ($v=\nabla\phi$)

$$\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=0$$

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Different forms of Wave equation

- Wave equation for Displacement potential ($u = \nabla\psi$)

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

- Wave equation in the presence of a source

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = f(r,t)$$

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Helmholtz equation

- Wave equation in the frequency domain.
- Obtained by taking the Fourier transform of the time domain wave equation.

$$[\nabla^2 + k^2(r)]\psi(r,\omega) = f(r,\omega) \quad \text{where} \quad k(r) = \frac{\omega}{c(r)}$$

- Not suitable to broadband applications due to the complexity of obtaining the inverse fourier transform.

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Solution of the wave equation

- Three dimensional, elliptical partial differential equation.
- No universal solution available.
- The solution depends on the following factors:
    - Dimensionality of the problem
    - Sound speed variations $c(r)$
    - Boundary conditions
    - Source-receiver geometry
    - frequency and bandwidth
- Optimum approach is the hybridization of analytical and numerical methods.

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Wave equation solution in different co-ordinate systems

- Cartesian coordinates

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\psi(x, y, z) = \begin{cases} Ae^{ik\cdot r} \\ Be^{-ik\cdot r} \end{cases}$$

where $k = (k_x, k_y, k_z)$ is the wave vector and A, B are arbitrary amplitudes

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Wave equation solution in different co-ordinate systems

- Cylindrical coordinates

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$$

- For uniform line source, the solution of the wave equations is

$$\psi(r) = \begin{cases} CH_0^{(1)}(kr) \\ DH_0^{(2)}(kr) \end{cases}$$

where $H_0^{(1)}, H_0^{(2)}$ are the hankel functions which can be represented in the asymptotic form as

$$H_0^{(1)}(kr) \simeq \sqrt{\frac{2}{\pi kr}}e^{i(kr-\pi/4)}$$

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- Spherical Co-ordinates

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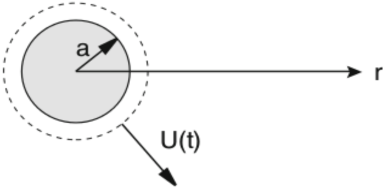
Source in an unbounded medium



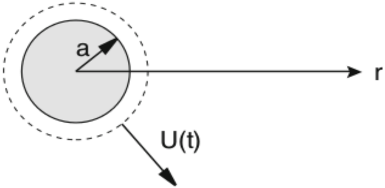
- Assuming an acoustic field in a homogeneous fluid due to a small sphere of radius a with a surface displacement given as

$$u_r(t,a) = U(t)$$

- In the homogeneous fluid, the field will be omni-directional, with the radial displacement

$$u_r = \frac{\partial \psi(r,t)}{\partial r}$$

Source in an unbounded medium



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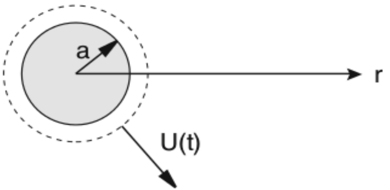

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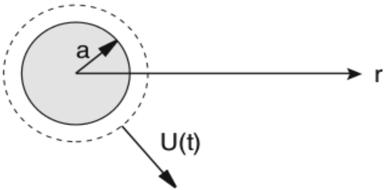

- where ψ is the displacement potential.
- Taking the fourier transform and applying the boundary condition, the solution the displacement field becomes

$$\psi(r) = -S_\omega \frac{e^{ikr}}{4\pi r}$$

where S_ω is the source strength.

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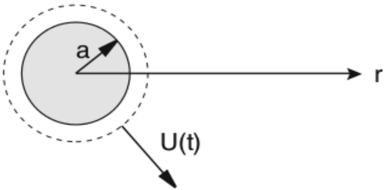

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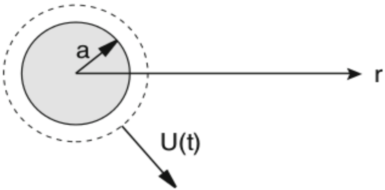

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Green Function

- Green function can be defined as the behavior of the channel. In frequency domain it is given by

$$g_\omega(r, r_0) = \frac{e^{ikR}}{4\pi R} \quad \text{where} \quad R = |r - r_0|$$

- Green function satisfies the inhomogeneous Helmholtz equation,

$$[\nabla^2 + k^2] g_\omega(r, r_0) = -\delta(r - r_0)$$

- The green function for the time domain wave equation is obtained by taking the Fourier transform

$$g_t(r, r_0) = \frac{\delta(R/c - t)}{4\pi R}$$

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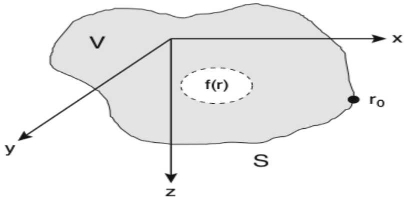
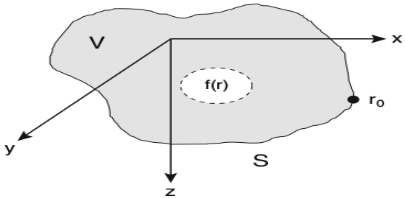


- In case of more realistic environment like a bounded media, the green function satisfies the Helmholtz equation by

$$[\nabla^2 + k^2]G_\omega(r, r_0) = \delta(r - r_0)$$
$$G_\omega(r, r_0) = g_\omega(r, r_0) + H_\omega(r)$$

- Using the green function for the bounded medium and doing some mathematical manipulations

Source in a bounded medium



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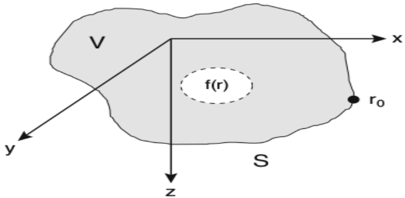

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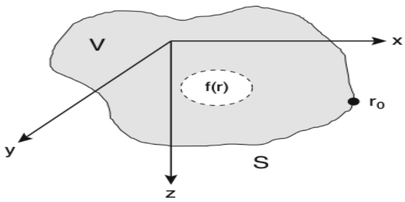

$$\psi(r) = \int_S \left[G_\omega(r, r_0) \frac{\partial \psi(r_0)}{\partial n_0} - \psi(r_0) \frac{\partial G_\omega(r, r_0)}{\partial n_0} \right] dS_0$$
$$- \int_V f(r_0) G_\omega(r, r_0) dV_0$$

where n_0 is the outward pointing normal on the surface.

- Difficult to attain the closed form solution.
- Make assumptions on the green function to simplify the expression.

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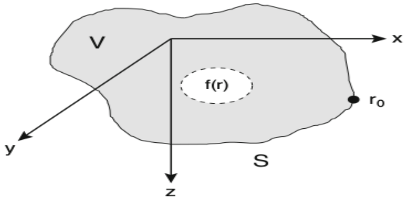

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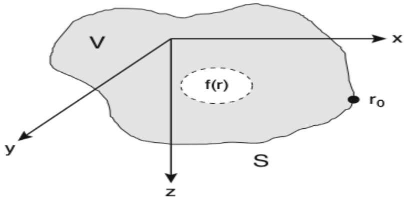
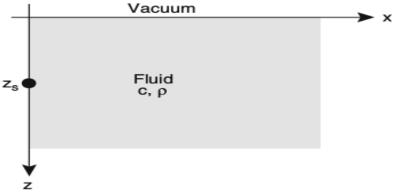


$$\psi(r) = \int_S \left[G_\omega(r, r_0) \frac{\partial \psi(r_0)}{\partial n_0} - \psi(r_0) \frac{\partial G_\omega(r, r_0)}{\partial n_0} \right] dS_0$$
$$- \int_V f(r_0) G_\omega(r, r_0) dV_0$$

where n_0 is the outward pointing normal on the surface.

- Difficult to attain the closed form solution.
- Make assumptions on the green function to simplify the expression.

Point source in fluid halfspace



- Assuming a point source is placed at $r_s = (x_s, y_s, z_s)$ and origin at the surface, we can replace the pressure release boundary condition by

$$\psi(r_0) = 0, \qquad r_0 = (x, y, 0)$$

- Using the green function for the bounded media we have

$$\psi(r) = \int_S G_\omega(r, r_0)\frac{\partial \psi(r_0)}{\partial n_0} - \int_V f(r_0) G_\omega(r, r_0) dV_0$$

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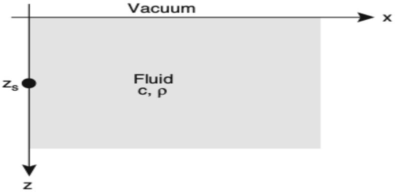

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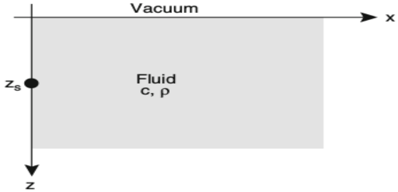

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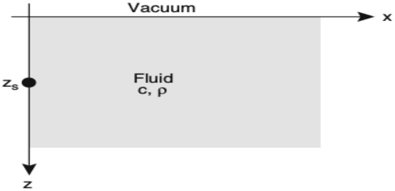
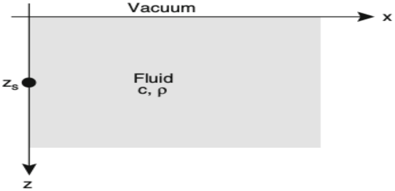


- where n_0 is the outward pointing normal on the surface.
- For a point source, the source field is given by

$$f(r_0) = S_\omega \delta(r_0 - r_s)$$

- In order to simplify, we can choose the green function such that $G_\omega(r, r_0) = 0$ then the displacement potential becomes

$$\psi(r) = -S_\omega G_\omega(r, r_s)$$



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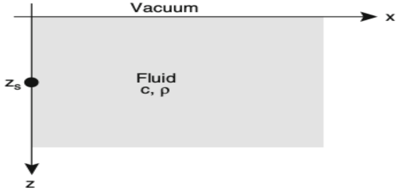

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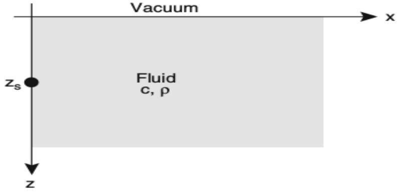

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Integral Transform Technique

- Applicable when both the coefficients of the Helmholtz equation and boundary conditions are independent of one or more space coordinates.
- Choice of the coordinate system is important.
- Boundary conditions control the choice of the coordinate system.

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Plane propagation problem

- Choosing the Cartesian coordinates and assuming an infinite line source along y-axis, the Helmholtz equation is reduced to two dimension, the range and the depth.

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2(z)\right]\psi(x,z) = S_\omega \delta(x)\delta(z-z_s)$$

- The boundary condition is given by

$$B[\psi(r)]_{z=z_n} = 0, \qquad n = 1,\ldots,N$$

- Taking the Fourier transform to obtain the depth separated equation and inserting the value of the displacement potential,

$$\left[\frac{d^2}{dz^2} + (k^2 - k_x^2)\right] G_\omega(k_x, z, z_s) = -\frac{\delta(z-z_s)}{2\pi}$$

- $G_\omega(k_x, z, z_s)$ is the depth dependent Green function, which is the superposition of the free field and homogeneous field.
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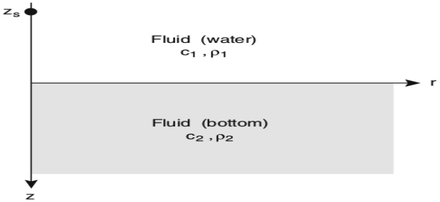
Reflection and Transmission



- Assuming the simplest of the bottom model as shown in the figure. The point source is located in the water column at depth $z = z_s$.
- For homogeneous medium, the solution of the equation is given as

$$H_\omega(k_r, z) = A^+(k_r)e^{ik_z z} + A^-(k_r)e^{-ik_z z}$$

- Where k_z is the vertical wavenumber, given by

$$k_z = \sqrt{k^2 - k_r^2}$$

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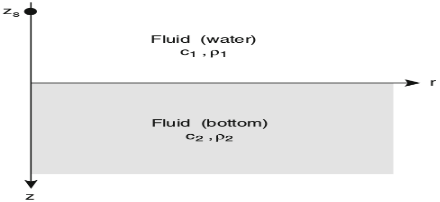

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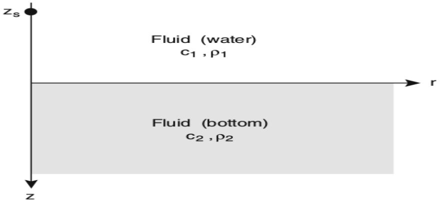

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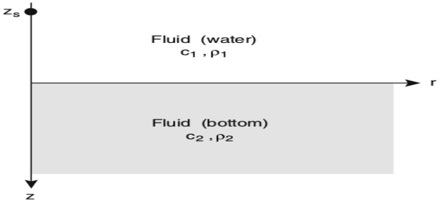

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- Using the radiation condition, the homogeneous solution in the upper halfspace is given by

$$H_{\omega,1}(k_r, z) = A_1^-(k_r)e^{-ik_{z,1}z}$$

- Similarly the solution in the lower halfspace will be

$$H_{\omega,2}(k_r, z) = A_2^+(k_r)e^{ik_{z,2}z}$$

- Both unknown amplitudes can be found using the boundary conditions of continuity of vertical displacement and continuity of pressure. The expression for the amplitudes is given by

$$A_1^- = \frac{\rho_2 k_{z,1} - \rho_1 k_{z,2}}{\rho_2 k_{z,1} + \rho_1 k_{z,2}} g_{\omega,1}(k_r, 0, z_s)$$

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Hard Bottom Vs Soft Bottom

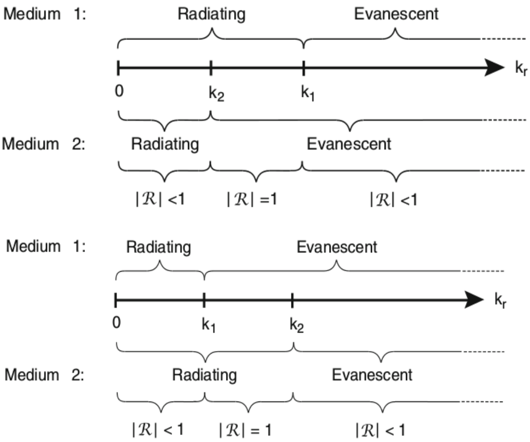


Figure : Spectral Domain for the hard bottom (top), soft bottom (bottom)

Hard Bottom Vs Soft Bottom

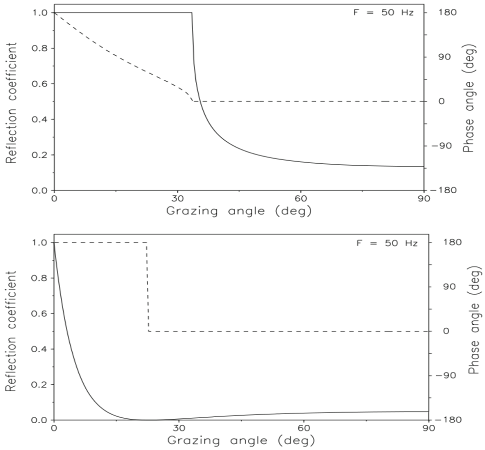
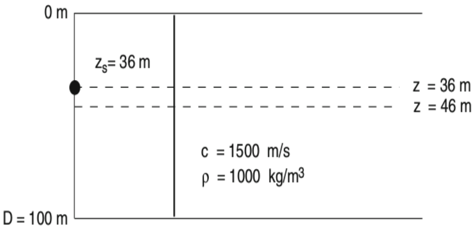
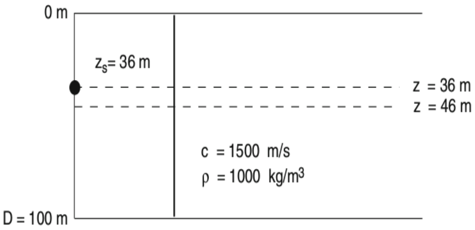


Figure : Reflection coefficient as a function of grazing angle for hard bottom (top), soft bottom (bottom). Solid curve: Magnitude. Dashed curve: Phase

Ideal fluid waveguide



- Assuming the simplest ocean waveguide with range independent, isovelocity water column and perfectly rigid boundaries as shown in the figure.
- Solution of the waveguide problem can be obtained by superposition principle.



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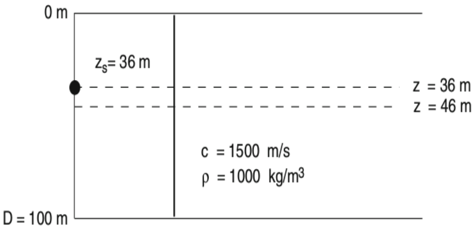

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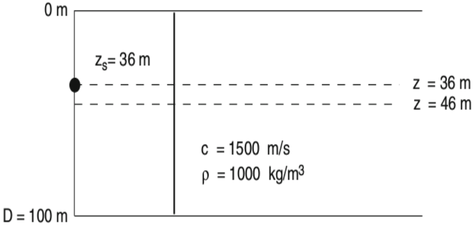

- The field produced at point $(0, z_s)$ in the absence of boundaries is given by

$$\psi(r, z) = -S_\omega \frac{e^{ikR}}{4\pi R}$$

- For solving the homogeneous equation which satisfies the boundary conditions, we can use two methods
  - Image Method.
  - Integral Transform solution.

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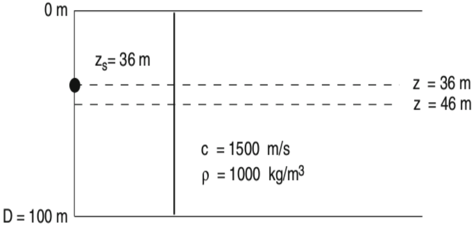

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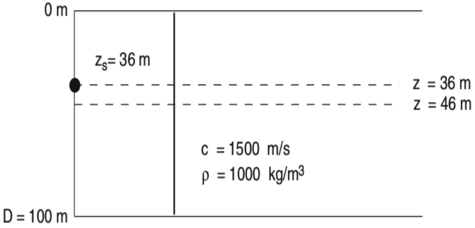

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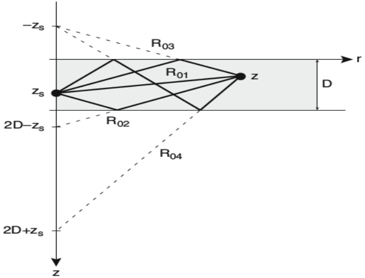

- In the waveguide problem, sound will be multiply reflected between the two boundaries and require an infinite number of image sources.
- Figure above shows the contribution from the physical source at depth z_s and the first three image sources, leading to the first four terms in the expression in the total field

$$\psi(r,z) \simeq \frac{-S_\omega}{4\pi}\left[\frac{e^{ikR_{01}}}{R_{01}} - \frac{e^{ikR_{02}}}{R_{02}} - \frac{e^{ikR_{03}}}{R_{03}} + \frac{e^{ikR_{04}}}{R_{04}}\right]$$

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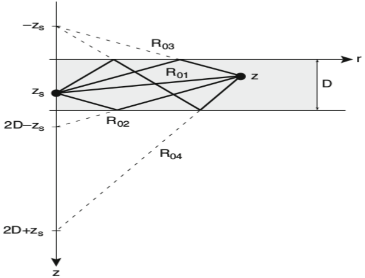

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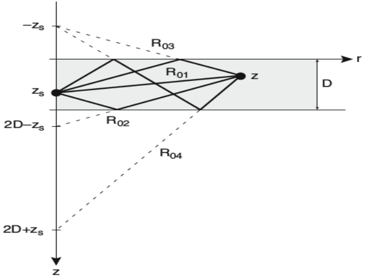

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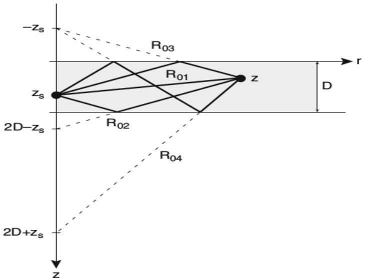
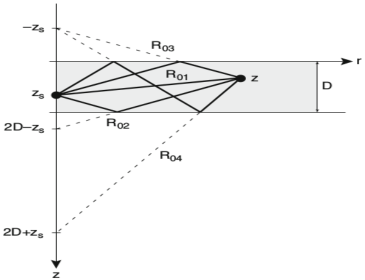


- where the negative sign corresponds to an odd number of reflections and the positive signs correspond to an even number of reflections.

- Expanding it to the total field, we have

$$\psi(r,z) = \frac{-S_\omega}{4\pi} \sum_{m=0}^{\infty} \left[\frac{e^{ikR_{m1}}}{R_{m1}} - \frac{e^{ikR_{m2}}}{R_{m2}} - \frac{e^{ikR_{m3}}}{R_{m3}} + \frac{e^{ikR_{m4}}}{R_{m4}} \right]$$



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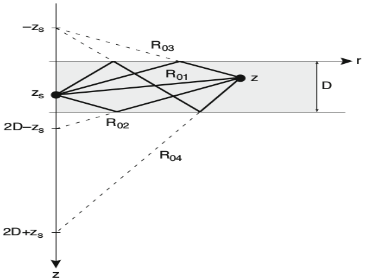

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- Computing the time domain green function, we have

$$g_t(r,z) = \frac{1}{4\pi}\sum_{m=0}^{\infty}\left(\frac{\delta(R_{m1}/c - t)}{R_{m1}} - \frac{\delta(R_{m2}/c - t)}{R_{m2}} - \frac{\delta(R_{m3}/c - t)}{R_{m3}} + \frac{\delta(R_{m4}/c - t)}{R_{m4}}\right)$$

- In order to obtain the received signal, the source function is convolved with the time domain green function.
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Integral Transform solution

- Using the integral transform technique, the total field is represented as

$$\psi(r,z) = \int_0^\infty \psi(k_r,z) J_0(k_r r) k_r dk_r$$

- Using the superposition principle and applying the boundary conditions, the free-field waveguide solution becomes

$$\psi(k_r,z) = -\frac{S_\omega}{4\pi} \begin{cases} \frac{sink_z z sink_z(D-z_s)}{k_z sink_z D}, z < z_s \\ \frac{sink_z z_s sink_z(D-z)}{k_z sink_z D}, z > z_s \end{cases}$$

- Using the relation between Hankel and Bassel function and doing some algebraic manipulations we have

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- This is the normal mode expansion of the field.
- The field remains the same even if the source and the receiver are interchanged
- Modal excitation is proportional to the amplitude of that particular mode at the source depth.
- Propagating Vs Evanescent modes.

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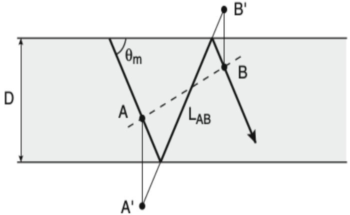
Relation between Rays and Modes



- A normal mode is the superposition of up and down going plane waves.

$$sin(k_{zm}z) = \frac{e^{ik_{zm}z} - e^{-ik_{zm}z}}{2i}$$

- Both of the waves are propagating at the grazing angles $\theta_m = arctan(k_{zm}/k_{rm})$. The propagation path is shown in the figure.
- The dashed line shows the common wavefront for the wave passing through points A and B. The distance between A and B is given by $L_{AB} = m\lambda$ where λ is the acoustic wavelength.
- The discrete wavenumbers are the points where multiple reflections of the plane wave are in phase, giving rise to resonance.

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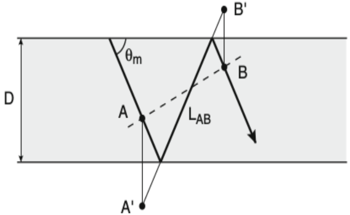

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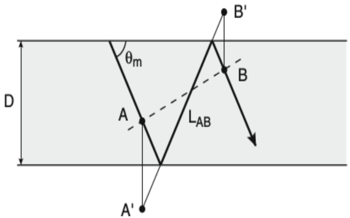

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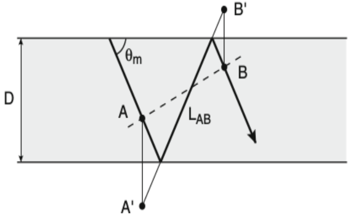

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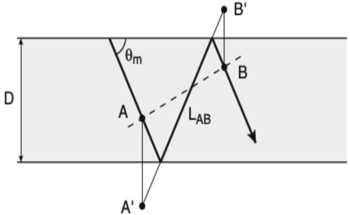

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Deep Ocean waveguide

- Depth variations and temporal variations of the sound speed profile.
- The scale of temporal variations is much larger than the vertical variations.
- Range independent environment can provide a realistic model of the deep ocean specially for the arctic environment.
- In the exact solution, the water column is divided in multiple layers and the wave equation is solved for each layer.
- WKB solution approximates the solution of the wave equation by amplitude and phase where both are functions of depth.

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